Image Denoising Utilizing the Scale-dependency in the Contourlet Domain

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***Abstract*—A new contourlet-based method is introduced for reducing noise in images corrupted by additive white Gaussian noise. This method takes into account the statistical dependencies among the contourlet coefficients of different scales. In view of this, a non-Gaussian multivariate distribution is proposed to capture the across-scale dependencies of the contourlet coefficients. This model is then exploited in a Bayesian maximum *a posteriori* estimator to restore the clean coefficients by deriving an efficient closed-form shrinkage function. Experimental results are performed to evaluate the performance of the proposed denoising method using typical noise-free images contaminated by simulated noise. The results show that the proposed method outperforms some of the state-of-the-art methods in terms of both the subjective and objective criteria.**

Keywords—*Contourlet transform, image denoising, Bayesian MAP estimator, multivariate distribution.*

I. INTRODUCTION

Image denoising is an important pre-processing step for image processing tasks. The problem of removing noise using multiscale transforms have been investigated in many works [1]-[5]. The contourlet transform (CT)-based approaches, either by using a simple thresholding scheme or the Bayesian approach, have led to a significant success in image denoising as compared to the earlier wavelet-based methods [6]. In the latter approach, a shrinkage function has been developed based on minimizing a Bayesian risk under the maximum *a posteriori* (MAP) criterion [1], [2]. The performance of the Bayesian MAP estimator depends on the accuracy of the prior for the contourlet coefficients. To the best of our knowledge, only a few statistical models have appeared in the literature for modeling the contourlet coefficients of an image, including the generalized Gaussian [7], normal inverse Gaussian (NIG) [8] and alpha-stable distributions [7], [9], [10]. It is known that the contourlet subband coefficients of an image have significantly non-Gaussian and heavy-tailed properties that are best described by heavy-tailed distributions [9].

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On the other hand, it has been shown in [6] that there exists a strong inter-scale dependency among the contourlet coefficients. In [6], a hidden Markov tree (HMT-CT) model has been proposed to capture such dependencies in an image denoising task. In [8], the NIG distribution has been used as a prior for contourlet coefficients in an image denoising task. It has been shown in [11] that the translation invariant CT provides better denoising results than the traditional one. In [12], a trivariate prior for the contourlet coefficients along with a non-local means filter has been proposed. In this work, in the expression for a new MAP estimation-based image denoising method, the contourlet subband coefficients are modeled by a non-Gaussian multivariate probability density function (PDF) to capture both the heavy-tailed properties of the contourlet coefficients of an image and their dependencies across scales. An efficient closed-form shrinkage function corresponding to this estimator is derived. The parameter estimation is performed with the assumption of the correlation between the variance of the neighboring coefficients in a square-shaped local neighborhood.

II. IMAGE DENOISING

It is known that the contourlet coefficients of an image have across scale dependencies with their parents and children [6], [7]. Fig. 1 depicts a parent-children relationship for three-scale contourlet decomposition with eight directions in each scale. This dependency plays an important role in the modeling of the contourlet coefficients. On the other hand, the contourlet coefficients of an image are non-Gaussian [7], [9], i.e., having large peaks around zero and tails heavier than that of a Gaussian PDF. In view of this, in this work, we propose using a non-Gaussian multivariate PDF to not only capture the heavy tails of the distribution of the contourlet coefficients but also to take into account their dependencies across scales. The PDF of the proposed non-Gaussian multivariate distribution is expressed as

$$P_x(x;\sigma,J) = \frac{K}{\left(\sqrt{2\pi\sigma}\right)^J}\exp\left(-\frac{\sqrt{(J+1)}}{\sigma}\sqrt{\left(x_1^2+x_2^2+\ldots+x_J^2\right)}\right) \tag{1}$$

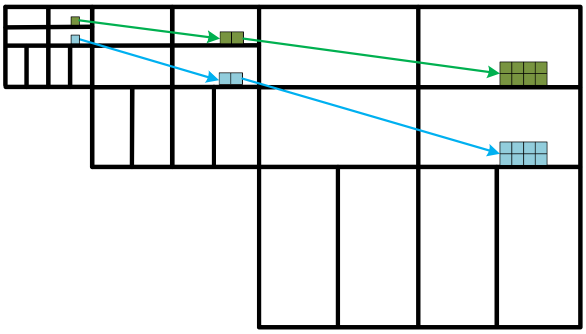

Fig. 1. Parent-children relationship for three-scale contourlet decomposition with eight directions in each scale.

where K is a constant to flexibly change the amplitude of the distribution, J is the dimensionality of the distribution, i.e., the number of scales, and σ denotes the standard deviation of the clean contourlet coefficients. Fig. 2 shows a possible configuration of the proposed non-Gaussian distribution across two finest scales of the contourlet coefficients.

III. BAYESIAN MAP ESTIMATOR

Let a noise-free image X be corrupted with an additive noise N which is assumed to be i.i.d. Gaussian with zero mean and standard deviation σ_η. The corresponding noisy image Y is given by $Y = X + N$. Suppose that a noisy image is decomposed to $j = 1,...,J$ scales and $d = 1,...,D$ directional subbands by the contourlet transform. Then, we have

$$y_j^d(m,n) = x_j^d(m,n) + \eta_j^d(m,n) \tag{2}$$

where $y_j^d(m,n)$, $x_j^d(m,n)$ and $\eta_j^d(m,n)$ denote the $(m,n)^{th}$ contourlet coefficient at scale j and direction d of the contourlet transform of Y, X and N, respectively. The purpose here is to estimate the noise-free coefficients

$$\begin{aligned}\hat{x}(y) &= \arg\max_x P_{x|y}(x \mid y) \\ &= \arg\max_x P_{y|x}(y \mid x) P_x(x) \\ &= \arg\max_x P_\eta(y - x) P_x(x)\end{aligned} \tag{3}$$

where $P_x(x)$ is the PDF of the contourlet coefficients of a noise-free image and $P_\eta(\eta)$ is the noise PDF which is given by

$$P_\eta(\eta) = \frac{1}{\left(\sqrt{2\pi}\sigma_\eta\right)^J} \exp\left(-\frac{\eta_1^2 + \eta_2^2 + ... + \eta_J^2}{2\sigma_\eta^2}\right) \tag{4}$$

If σ_η is unknown, it may be estimated by applying the robust median absolute deviation method [13] in the finest subband of the observed noisy coefficients.

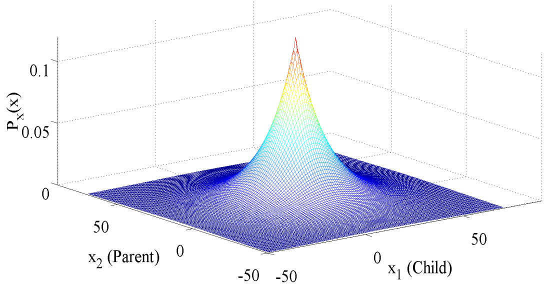

Fig. 2. Proposed non-Gaussian PDF for contourlet coefficients across two finest scales, $J = 2$.

The parameter σ of the noise-free coefficients can be estimated by using

$$\hat{\sigma}_j^2(m,n) = \max\left(\sum_{\substack{i \in W(n-\frac{l-1}{2},\, n+\frac{l-1}{2}) \\ m>n}} \frac{y_j^2(i)}{l^2} - \sigma_{\eta,j}^2, 0\right) \tag{5}$$

where W is a square-shaped window of size $l \times l$ applied to estimate the variance of each signal coefficient in a given scale j. To obtain the MAP estimate, after inserting (4) into (3), the derivative of the logarithm of the argument in (3) is set to zero resulting in

$$\frac{\hat{x}_i - y_i}{\sigma_\eta^2} + \frac{\partial}{\partial x_i}\left(-\ln\left(P_x(x)\right)\right) = 0, \qquad i = 1,2,...,j \tag{6}$$

To ensure the consistency of the signs of $\hat{x}_i$ and y_i, an approximate bounded solution of (6) is obtained as

$$\begin{aligned}\hat{x}_i(y_i) &= \mathrm{sign}(y_i).\left(|y_i| - \sigma_\eta^2 |M|\right)_+ \\ M &= \frac{\left(\frac{\sqrt{J+1}}{\sigma}\right)}{\sqrt{y_1^2 + y_2^2 + ... + y_J^2}}\end{aligned} \tag{7}$$

where $(z)_+ = \max(z,0)$. Thus, the denoising method can be summarized as follows

Algorithm: CT-based image denoising with multivariate prior

1. Perform the contourlet transform decomposition on the noisy image.
2. Estimate the standard deviations of the proposed multivariate distribution and noise.
3. Estimate the noise-free coefficients $\hat{x}_i^d(m,n)$ using the closed-form Bayesian MAP estimator in (7).
4. Apply the inverse contourlet transform to the estimated coefficients and restore the denoised image.

IV. SIMULATION RESULTS

We perform experiments on a set of grayscale images, each of size 512×512, to evaluate the performance of the proposed denoising method and to compare the results to that obtained by Adaptive-shrink [1], Bayes-shrink [2], HMT-W [4], GSM [5], NIG-CT [8], STICT [11], Visu-shrink [13], NIG-NSCT [14], SURE-shrink [15] and MMSE-MAP [16]. The noisy image is first decomposed by the contourlet transform with 9-7 biorthogonal filters to five scales and eight directions in each scale. It is found experimentally that $l = 5$ leads to an optimum denoising result. Since the contourlet transform is shift-variant, in order to overcome the possible pseudo-Gibbs phenomenon in the neighborhood of discontinuities, the cycle spinning mode [17] is performed on noisy images. Table I gives the peak signal-to-noise-ratio (PSNR) values obtained by using various denoising methods for two of the test images, namely, *Lena* and *Boat*. It can be seen from this table that the proposed method using the multivariate prior yields better results than some of the existing methods. In particular, for *Lena* image, there is an increase on PSNR value achieved by the proposed method in low levels of noise, σ_η varying from 10 to 15. In the case of *Boat* image, the proposed method yields better results at all cases. Table II gives the PSNR values of the proposed method, HMT-W and HMT-CT, σ_η varying from 30 to 50. It is seen from this table that the proposed method provides higher PSNR values than other methods. Fig. 3 illustrates the denoised results of the cropped *Boat* image at $\sigma_\eta = 25$ along with the mean structural similarity (MSSIM) index. It can be seen from this figure that the proposed denoising method outperforms some of the existing methods in terms of the visual quality of the denoised image.

V. CONCLUSION

In this work, a new contourlet domain image denoising method has been proposed. We have developed a statistical model for the contourlet coefficients of images using a non-Gaussian multivariate distribution that takes into account both their heavy-tailed property and dependencies across scales. To estimate the noise-free coefficients, the noisy image has been decomposed into various scales and directional subbands via the contourlet transform. A Bayesian MAP estimator utilizing the proposed prior has been developed. The noise in all the detail subbands has been removed by deriving an efficient closed-form shrinkage function. Experiments have been carried out to compare the performance of the proposed method to that of some of the existing methods. The results have shown that the proposed denoising method outperforms some of the existing methods in terms of the PSNR values and provides denoised images with high visual quality.

TABLE I
PSNR values (in dB) obtained using various denoising methods for *Lena* and *Boat* images.

	Standard deviation			
Method	**10**	**15**	**20**	**25**
Lena				
Noisy image	28.12	24.62	22.13	20.16
Proposed	**34.75**	**32.53**	31.18	30.13
Visu-shrink(soft) [13]	28.70	27.40	26.59	26.04
Visu-shrink(hard) [13]	30.65	28.89	27.76	27.02
Adaptive-shrink [1]	33.48	32.36	31.04	30.04
Bayes-shrink [2]	33.29	31.38	30.14	29.19
HMT-W [4]	33.81	31.73	30.36	29.21
NIG-CT [8]	32.46	30.79	29.14	28.02
STICT [11]	34.67	-	**31.52**	-
SURE-shrink [15]	33.42	31.50	30.17	29.18
NIG-NSCT [14]	34.09	-	31.18	-
GSM [5]	34.24	32.35	31.03	**30.23**
MMSE-MAP [16]	34.32	-	31.09	-
Boat				
Noisy image	28.13	24.61	22.11	20.17
Proposed	**33.78**	**31.07**	**29.87**	**28.56**
Visu-shrink(soft)[13]	26.64	25.34	24.59	24.06
Visu-shrink(hard) [13]	28.61	26.90	25.82	25.03
Adaptive-shrink [1]	31.93	30.01	28.37	27.23
Bayes-shrink [2]	31.77	29.84	28.45	27.37
HMT-W [4]	32.25	30.28	28.81	27.65
NIG-CT [8]	31.79	30.10	28.76	27.43
STICT [11]	33.31	-	29.79	-
SURE-shrink [15]	31.83	29.88	28.55	27.50
NIG-NSCT [14]	30.95	-	28.42	-
GSM [5]	32.39	30.41	29.03	27.99
MMSE-MAP [16]	32.43	-	28.94	-

TABLE II
PSNR values obtained using proposed denoising method and that obtained using the methods in [4] and [6].

	Standard deviation		
Method	**30**	**40**	**50**
Lena			
Noisy image	18.88	16.53	14.63
Proposed	**29.65**	**28.31**	**27.11**
HMT-W [4]	28.35	27.21	25.89
HMT-CT [6]	28.18	27.00	26.04

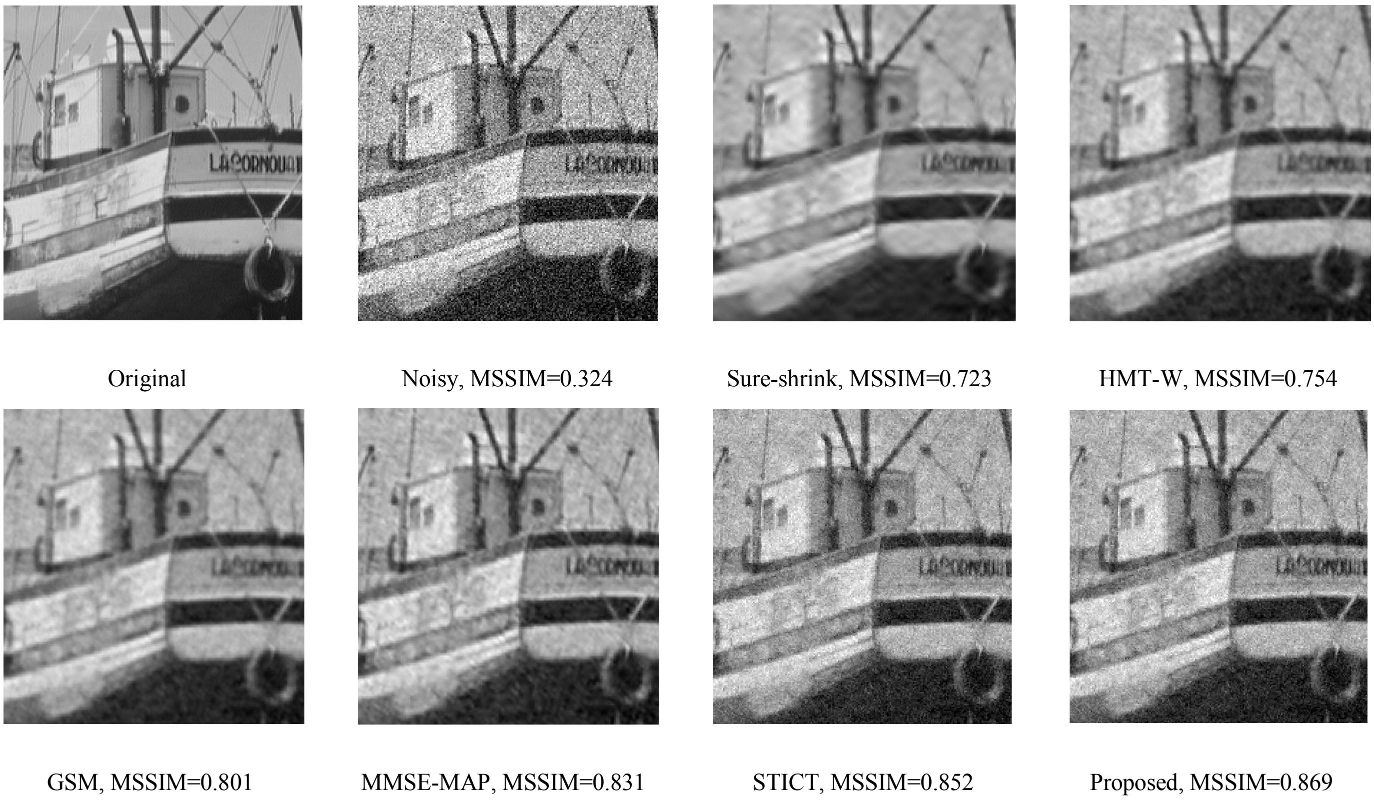

Fig. 3. Visual and MSSIM quality comparisons of various denoising methods for the cropped *Boat* image with $\sigma_\eta = 25$.

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