# Ultrasound Image Despeckling in the Contourlet Domain Using the Cauchy Prior

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***Abstract*— Speckle noise reduction is a prerequisite task in images captured by ultrasonography systems due to their inherent noisy nature. In this work, we propose a new despeckling method in the contourlet domain using the Cauchy prior. The multiplicative speckle noise is first transferred to an additive one using a logarithmic transform. The logarithmically-transformed contourlet coefficients of the image and noise are assumed to be the Cauchy and Maxwell distributions, respectively. In order to estimate the noise-free contourlet coefficients, an efficient closed-form Bayesian maximum *a posteriori* estimator is developed. Simulations are carried out to evaluate the performance of the proposed despeckling method by using the synthetically-speckled and real ultrasound images. It is shown that the proposed method outperforms several existing techniques in terms of the signal-to-noise ratio and is able to preserve the diagnostically significant details of the ultrasound images.**

**Keywords—** ***Ultrasound image despeckling, contourlet transform, Cauchy distribution, Bayesian MAP estimator.***

## I. INTRODUCTION

It is known that the ultrasonography systems are intrinsically contaminated by the speckle noise. Speckle noise considerably degrades the image quality and obscures making a diagnosis. Despeckling of ultrasound images is an inevitable preprocessing procedure in order to avoid any negative impact on diagnostic task. Several techniques, from spatial filters for e.g., median, and Wiener filters to frequency domain filters for e.g., homomorphic filter and wavelet shrinkage, have been proposed to reduce the speckle noise [1], [2]. Spatial domain filters, however, suppress the speckle noise at the expense of blurring many important image details. The frequency domain techniques, on the other hand, have shown to preserve more details such as anatomical boundaries in the image. The homomorphic filter-based method has been proposed to convert the multiplicative speckle noise to an additive one using a logarithmic transformation [2]. This method has been combined by the Bayesian estimators to outperform classical linear processors and simple thresholding estimators in removing speckle noise from ultrasound images. In Bayesian methods, a suitable probability density function (PDF) is utilized as a prior model for characterizing the log-transformed coefficients [3], [4]. In [5], a Bayesian MAP estimator is developed by using the Gaussian PDF for modeling the signal coefficients, and the Rayleigh PDF for modeling the log-transformed speckle noise. In [6], a homomorphic method for simultaneous compression and denoising of ultrasound images has been developed by modeling the coefficients using the generalized Gaussian PDF. In [7], a homomorphic method has been proposed in the wavelet domain. In [8], a multiscale-based method for despeckling the ultrasound images has been proposed by employing a generalized likelihood ratio.

The contourlet transform has been shown to provide significant noise reduction in comparison to that provided by the earlier wavelet-based methods [9]. This is due to its flexible directional decomposability in each scale as compared to the wavelet transform in representing smooth contour details in images [9]. The contourlet subband coefficients of an image have been shown to be highly non-Gaussian and heavy-tailed and thus, modeled by the non-Gaussian distributions such as the Cauchy PDF [10], [11].

It should be noted that the performance of the Bayesian estimators depend significantly on the accuracy of the models assumed for the prior PDFs of the log-transformed ultrasound image and noise. In many works, the noise PDF has been considered to follow the Gamma PDF. However, after logarithmic transformation made by the homomorphic filter the speckle noise is converted to an additive one [2].

In this work, a new contourlet domain method for despeckling medical ultrasound images is proposed. The contourlet coefficients of the log-transformed reflectivity are modeled by the Cauchy PDF while those of the log-transformed noise are assumed to follow the Maxwell PDF. A closed-form Bayesian maximum *a posteriori* (MAP) estimator is derived by using the Cauchy distribution that exploits the statistics of the contourlet coefficients. A maximum likelihood method is used for parameter estimation. Simulations are conducted using synthetically-speckled and real ultrasound images to evaluate the performance of the proposed method and to compare it with that of some of the existing techniques.

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## II. SIGNAL MODEL

In noise removal task, it is known that the contourlet transform has led to a better performance than other multiscale transforms do [9]. In this work, in order to despeckle the inherit noise of the ultrasound images, we propose using the statistical properties of both the contourlet coefficients and noise to derive an efficient closed-form estimator for noise removal. It has been shown that the distribution of the contourlet coefficients of images can be suitably modeled by the Cauchy distribution [10], [11]. The zero-mean Cauchy PDF is given by

$$P_x(x) = \frac{\gamma}{\pi(\gamma^2 + x^2)} \tag{1}$$

where $\gamma > 0$ is the dispersion parameter. The log-likelihood function for the Cauchy distribution is given by

$$\log L(\gamma; x_1, x_2, ..., x_n) = -n\log(\pi\gamma) - \sum_{i=1}^{n} \log\left(1 + \left(\frac{x_i}{\gamma}\right)^2\right) \tag{2}$$

In order to estimate the dispersion parameter, we maximize (2) with respect to $\gamma$ resulting in

$$\sum_{i=1}^{N} \frac{\gamma^2}{\gamma^2 + x_i^2} - \frac{n}{2} = 0 \tag{3}$$

which leads to an iterative estimator given by

$$\hat{\gamma}_{(k+1)} = \sqrt{\frac{n}{2} \frac{1}{\sum_{i=1}^{N} \frac{1}{\hat{\gamma}_{(k)}^2 + x_i^2}}} \tag{4}$$

## III. PROPOSED DESPECKLING METHOD

Since the speckle noise model for ultrasound images is considered to be multiplicative, the observed output of the ultrasound imaginary system can be defined as

$$f_{i,j} = g_{i,j} n_{i,j} \tag{5}$$

where $f_{i,j}$ denotes the $(i,j)$ th noisy pixel in an ultrasound image, $g_{i,j}$ the corresponding noise-free pixel and $n_{i,j}$ the multiplicative speckle component. With log-transformation, (5) becomes

$$Y_{i,j} = X_{i,j} + N_{i,j} \tag{6}$$

where $Y = \ln(f)$, $X = \ln(g)$ and $N = \ln(n)$.

Suppose that a noisy image is decomposed to $j = 1,...,J$ scales and $d = 1,...,D$ directional subbands by the contourlet transform. Then, we have

$$y_j^d(m,n) = x_j^d(m,n) + \eta_j^d(m,n) \tag{7}$$

where $y_j^d(m,n)$, $x_j^d(m,n)$ and $\eta_j^d(m,n)$ denote the $(m,n)^{th}$ contourlet coefficient at scale $j$ and direction $d$ of the contourlet transform of $Y$, $X$ and $N$, respectively. Despeckling is based on estimating the noise-free coefficients $x$ as a function of the noisy observations $y$. To this end, a Bayesian MAP estimator is developed through modeling the contourlet coefficients of a noisy image by the Cauchy PDF. The Bayesian MAP estimator of $x$, given noisy observation $y$, can be derived as

$$\begin{aligned} \hat{x}(y) &= \arg\max_x P_{x|y}(x \mid y) \\ \hat{x}(y) &= \arg\max_x P_{y|x}(y \mid x) P_x(x) \end{aligned} \tag{8}$$

where $P_x(x)$ is the PDF of the contourlet coefficients of a noise-free image. Then, (8) can be rewritten as

$$\hat{x}(y) = \arg\max_x P_\eta(y - x) P_x(x) \tag{9}$$

where $P_\eta(\eta)$ is the noise PDF. In the proposed despeckling method, the logarithmically-transformed speckle noise is assumed to be zero-mean, independent and identically distributed by the Maxwell distribution that its PDF is given by

$$P_\eta(\eta) = \sqrt{\frac{2}{\pi}} \frac{\eta^2}{v^3} \exp\left(-\frac{\eta^2}{2v^2}\right) \tag{10}$$

where $v = \frac{\sigma_\eta}{\sqrt{3}}$. The noise standard deviation $\sigma_\eta$ can be estimated by applying the robust median absolute deviation method [12] in the finest subband of the observed noisy coefficients. To obtain the MAP estimate, after inserting (1) and (10) into (9), the derivative of the logarithm of the argument in (9) is set to zero resulting in

$$\frac{\partial}{\partial y}\left(\ln\left(P_\eta(y - x)\right)\right) + \frac{\partial}{\partial y}\left(\ln\left(P_x(y)\right)\right) = 0 \tag{11}$$

After some manipulations, the Bayesian MAP estimator is derived as a root of the following quadratic equation

$$\hat{x}^4 + a\hat{x}^3 + b\hat{x}^2 + c\hat{x} + d = 0 \tag{12}$$

where

$$a = -2y, \qquad b = \gamma^2 + y^2$$

$$c = -2y\gamma^2 - \frac{2}{3}y\sigma_\eta^2, \qquad d = -\frac{2\sigma_\eta^2\gamma^2}{3} + \gamma^2 y^2 \tag{13}$$

The quadratic equation in (13) can be solved by using Ferrari's method [13] as

$$\left(\hat{x}^2 + \frac{1}{2}a\hat{x} + \frac{1}{2}t_0\right)^2 = \left(\sqrt{\hat{x}t_0 + \frac{1}{4}a^2 - b} \pm \sqrt{\frac{1}{4}t_0^2 - d}\right)^2 \tag{14}$$

By solving two quadratic equations in (14), $\hat{x}$ is found as

$$\begin{aligned}\hat{x} = &-\frac{a}{4} \pm \frac{1}{2}\sqrt{\frac{a^2}{4} - b + t_0} \\ &+ \frac{1}{2}\sqrt{\left(\frac{a}{2} \pm \sqrt{\frac{a^2}{4} - b + t_0}\right)^2 - 2t_0 \pm \sqrt{\frac{t_0^2}{4} - d}}\end{aligned} \tag{15}$$

where $t_0$ is a root of the cubic equation

$$t^3 - bt^2 + (ac - 4d)t + (4bd - a^2d - c^2) = 0 \tag{16}$$

as

$$t_0 = \sqrt[3]{-\frac{q}{2} + \sqrt{D}} + \sqrt[3]{-\frac{q}{2} - \sqrt{D}} + \frac{b}{3} \tag{17}$$

where

$$\begin{aligned} q &= \frac{b(ac - 4d)}{3} - \frac{2b^3}{27} - a^2d - c^2 + 4bd \\ D &= \frac{(ac - 4d - \frac{b^3}{3})}{27} - \frac{q^3}{4} \end{aligned} \tag{18}$$

It is to be noted that there is no more than one root in (18) that lies between 0 and $y$.

## IV. SIMULATION RESULTS

Simulations are carried out to evaluate the performance of the proposed method using synthetically-speckled and real ultrasound images. The results are compared to those obtained by using some of the existing speckle methods, namely median filtering, CW-Gaussian [2], CT-SNIG [3], W-GG [5], soft-thresholding [12] and Bayes-shrink [14]. The real speckled images are obtained from [15]. The synthetically-speckled images are obtained by corrupting the standard images of size 256 × 256 pixels obtained from [16] with speckle noise having various standard deviations. It should be noted that the contourlet transform is a shift-variant transform. Thus, in order to overcome the possible pseudo-Gibbs phenomenon in the neighborhood of discontinuities, in the proposed despeckling method, the cycle spinning [17] is performed on the observed noisy image. The log-transformed noisy image is decomposed, using the contourlet transform, into three scales with eight directions in each scale. The coefficients in all the detail subbands are despeckled using the proposed MAP estimator in (12). In order to quantify the performance improvement, the signal-to-noise ratio (SNR) is computed between the synthetically-speckled and denoised images. Table I gives the SNR values for two synthetically-speckled images, namely, *Lena* and *Barbara*, obtained using various despeckling methods. It is seen from this table that proposed despeckling method gives higher values of SNR compared to that provided by the other methods.

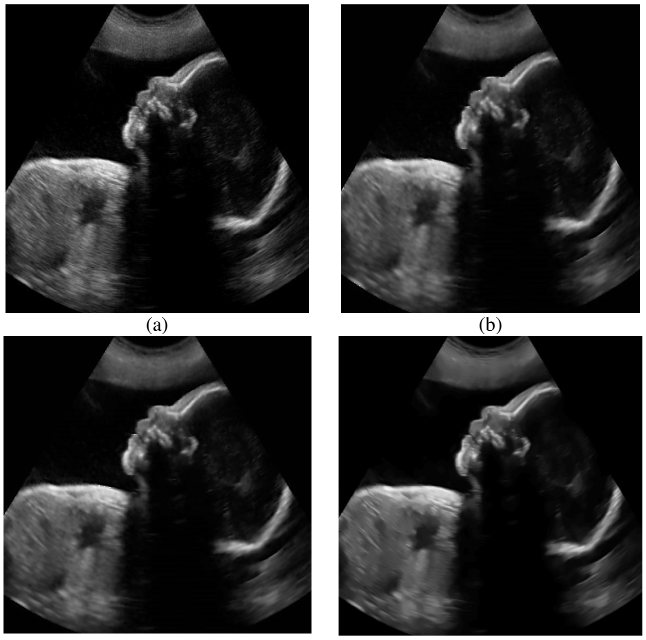
In order to subjectively evaluate the performance of the proposed despeckling method, Figs. 1 and 2 illustrate the real noisy ultrasound images as well as the despeckled ones obtained using the methods in [2], [3] and the proposed method. It can be observed from these figures that the proposed method not only provides an effective speckle suppression, but also preserves the diagnostically important details.

Table I. SNR values in dB obtained using various methods for two of the synthetically-speckled images, namely, *Lena* and *Barbara*.

| | **Standard deviation** | | | |
|---|---|---|---|---|
| | **0.5** | **0.6** | **0.7** | **0.8** |
| Synthetically-speckled Lena | | | | |
| **Proposed** | **18.25** | **17.46** | **16.70** | **15.65** |
| **Median filter** | 11.83 | 10.56 | 9.78 | 8.93 |
| **CW-NIG [2]** | 13.00 | 12.65 | 12.29 | 12.02 |
| **Bayes-shrink [14]** | 14.12 | 12.70 | 11.29 | 10.11 |
| **CT-SNIG [3]** | 16.60 | 15.43 | 14.60 | 13.74 |
| **Soft-thresholding [12]** | 12.47 | 11.28 | 10.37 | 9.56 |
| **W-GG [5]** | 16.03 | 15.14 | 14.21 | 13.19 |
| Synthetically-speckled Barbara | | | | |
| **Proposed** | **17.31** | **16.60** | **15.46** | **14.29** |
| **Median filter** | 8.63 | 7.78 | 6.57 | 5.61 |
| **CW-NIG [2]** | 9.57 | 9.40 | 9.18 | 9.70 |
| **Bayes-shrink [14]** | 12.28 | 10.72 | 9.50 | 8.35 |
| **CT-SNIG [3]** | 15.27 | 14.08 | 13.04 | 12.10 |
| **Soft-thresholding [12]** | 9.11 | 8.46 | 7.23 | 6.91 |
| **W-GG [5]** | 15.30 | 14.01 | 12.89 | 11.95 |

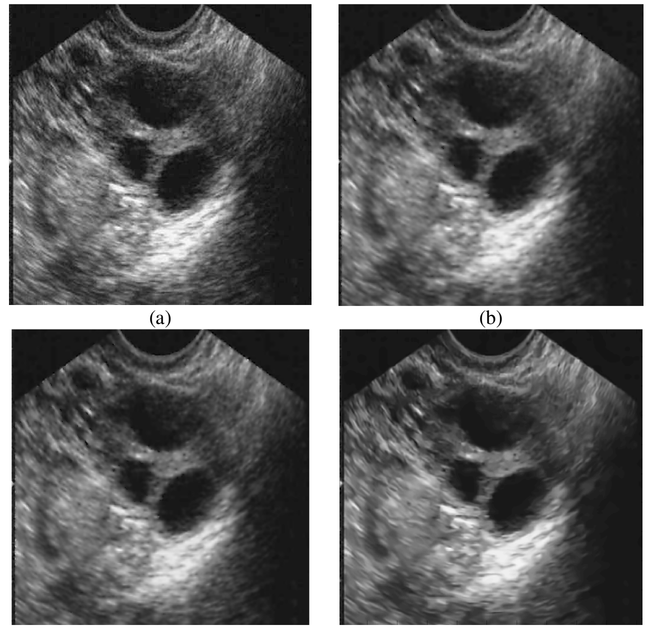
## V. CONCLUSION

In this work, a new despeckling method for ultrasound images in the contourlet domain has been proposed. The noisy ultrasound image is first logarithmically transformed



(a) (b) (c) (d)

Fig. 1. (a) Noisy ultrasound image, (b)-(d) Despeckled images obtained using the methods in [2], [3] and the proposed method, respectively.



(a) (b) (c) (d)

Fig. 2. (a) Noisy ultrasound image, (b)-(d) Despeckled images obtained using the methods in [2], [3] and the proposed method, respectively.

to convert the multiplicative speckle noise to an additive form. It is then decomposed into various scales and directional subbands via contourlet transform. The noise in all detail subbands is removed by an efficient closed-form Bayesian MAP estimator using the Cauchy prior. Experiments have been carried out to compare the performance of the proposed method with that of the some of the existing methods. The simulation results have shown that the proposed despeckling method outperforms these other methods in terms of SNR and provides superior visual quality denoised images by preserving diagnostically significant details.

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